RESEARCH DEPARTMENT

THE POWER LOSS FROM OPEN-WIRE HIGH-FREQUENCY TRANSMISSION LINES DUE TO SUPPORT STRUCTURES

Report No. E-079

(1962/52)

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

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SUMMARY

The balanced open-wire transmission lines used at high-frequency transmitting stations are subject to losses due to the currents induced in the supporting structures. This report considers the magnitude of these losses in the cases of steel and reinforced concrete support structures. A method for calculating the loss is presented and it is shown that significant power is lost only when the structure is electrically resonant.

1. INTRODUCTION

At high-frequency (h.f.) transmitting stations, balanced open-wire transmission lines are used to convey r.f. power from the transmitters to the switching stations and thence to the aerial arrays. The attenuation of these lines due to copper losses, insulator losses, earth currents and radiation has been investigated, and quantitative assessments of these losses, based mainly on measurements, are available.[1] An additional loss occurs, however, because the electromagnetic fields surrounding a line induce currents in the support structures and these currents radiate power; the available measured figures[1] refer to conditions in which this loss is negligible. Fig. 1 shows the two types of support structure used at h.f. transmitting stations. It is shown in Section 2 of this report that the part of the support structure which is most closely coupled with the transmission line is the horizontal member. In the arrangement shown in Fig. 1(a) the horizontal member is small compared with a wavelength and little power is therefore re-radiated; the principal effect of such a support is to introduce a small discontinuity in the line. In the structure shown in Fig. 1(b), however, the cross-member may be of the order of $\lambda/2$ in length at the higher frequencies so that the support may cause appreciable re-radiation as well as introducing a mismatch. Only the type of structure shown in Fig. 1(b) is therefore considered in detail and formulae for the loss are derived in Section 2.

Although existing designs of support structure are entirely metallic, it has been proposed that reinforced concrete structures should be used in future. The losses occurring in this case are therefore considered, and are discussed in Section 3.

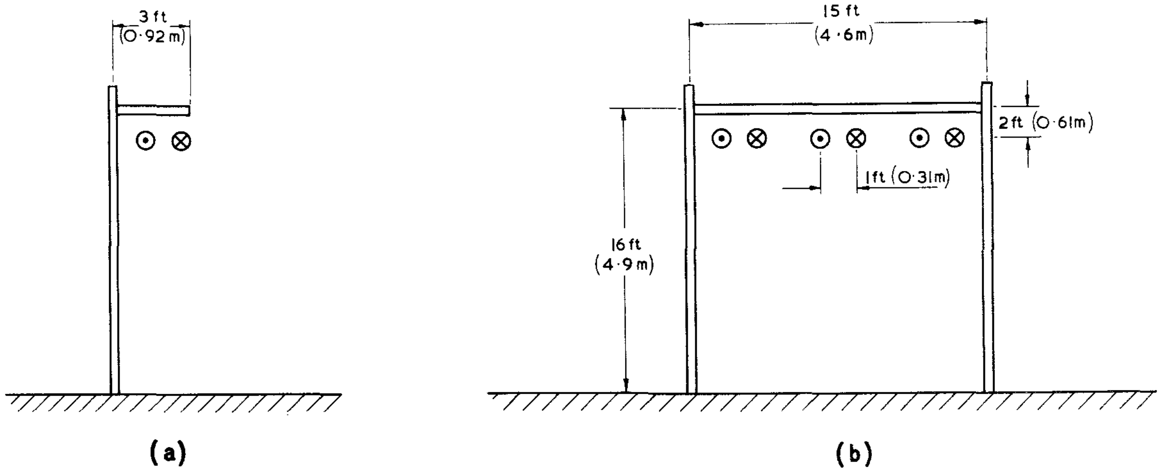

Fig. 1 - Diagrammatic representation of typical support structures

2. RE-RADIATION FROM METAL SUPPORT STRUCTURES

2.1 General Considerations

To facilitate analysis, a metal support structure can be regarded as being cut at a convenient point and provided with terminals, as shown in the upper part of Fig. 2(b); these would be short circuited when the normal behaviour of the structure is considered. The coupling between a transmission line and a metallic support structure may therefore be represented by a mutual impedance Z_M such that if the current in the transmission line is I_L (Z_0 being the characteristic impedance of the line), a voltage $I_L Z_M$ is induced at the terminals of the support structure. The self impedance and radiation resistance of the support structure measured at its terminals are designated Z_s and R_s respectively and thus, when Z_M is small compared with Z_0, the fraction of the transmitted power which is re-radiated by the support structure is:

$$\frac{|Z_M|^2 R_s}{|Z_s|^2 Z_0} \tag{1}$$

It can be deduced from expression (1) that, unless the support structure is resonant, very little power will be lost because $|Z_s|^2$ will be large compared with R_s^2. Maximum re-radiation will therefore occur when $Z_s = R_s$ and in this case expression (1) becomes:

$$\frac{|Z_M|^2}{R_s Z_0} \tag{2}$$

It is possible in practice that the support structures will be of sufficient size to resonate, and to determine an upper limit for the loss it is therefore necessary to find Z_M and R_s under resonant conditions. For small changes in frequency near resonance Z_M and R_s will not change greatly and hence it is necessary to know only Z_s as a function of frequency to determine the power radiated at frequencies near resonance. Further, Z_s may be estimated from the physical dimensions of the structure.

It has been shown[2] that the mutual impedance between two systems of conductors (1) and (2) whose cross sections are small compared with a wavelength and which are coupled electromagnetically is given by the line integral:

$$-\frac{1}{I_1 I_2} \int E_1(x)\ I_2(x)\ dx \tag{3}$$

where $E_1(x)$ is the electric field tangential to system (2) due to a current I_1 in system (1) when system (2) is removed and $I_2(x)$ is the current in system (2) due to an input current I_2 in system (2) when system (1) is open circuited, x being the distance along system (2) from the input terminals. Expression (3) is particularly simple to evaluate if open-circuiting the terminals of system (1) is equivalent to removing it, or alternatively, if the mutual impedance is small.

In the case considered, the two systems are the transmission line and the support structure and investigation shows that the current distribution on the support structure is virtually independent of the presence of the line because the magnitude of the mutual impedance is small. Thus if a current $I(x)$ flows in the support structure:

$$Z_M = -\frac{1}{I_L I_s} \int_{-L_1}^{L_1} E(x)\ I(x)\ dx \tag{4}$$

Expression (4) is evaluated for cases of practical interest in Section 2.2 and the remaining unknown in expression (2), R_s, is estimated in Section 2.3. For convenience the terminals are assumed to be situated at the centre of the horizontal member of the support structure.

2.2 The Mutual Impedance between the Transmission Line and a Support Structure

The type of support structure considered is that shown in Fig. 1(b) and it can be shown that the mutual impedance between line and structure is greater for the line in the centre than it is for the other two. The essentials of the system are therefore as shown in Fig. 2(a) but it is convenient to replace the ground plane by the equivalent image as shown in Fig. 2(b).

For simplicity the line is assumed to have only two conductors and the modifications necessary for multiconductor lines are considered in Section 4.

2.2.1 The Electromagnetic Fields due to the Transmission Line

In typical h.f. transmission lines, the ratio of the conductor spacing to the conductor radius is greater than ten and hence the current distribution round the perimeter of each conductor is substantially uniform. The total field may therefore be taken as the vector sum of the fields produced by each conductor in isolation.

It has been shown[3] that for a single conductor the only significant component of electric field is radial and has a value of 60 I_L/r_1, r_1 being the distance

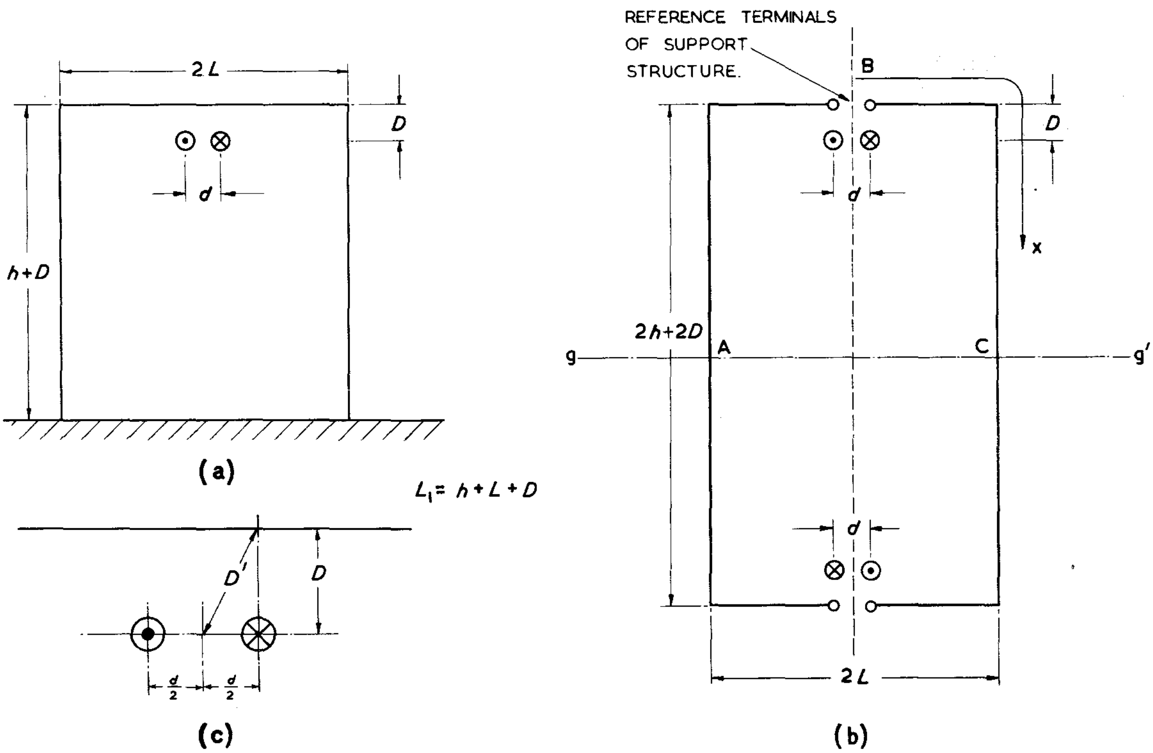

Fig. 2 - Essential features of a line coupled to a support structure

from the conductor to the point considered. Fig. 3 shows how the resultant electric field E_r at two points P_1 and P_2 may be found. It can be seen from Fig. 3 that the component of E_{r1} tangential to the horizontal member y y' is much greater than the component of E_{r2} tangential to the vertical member z z' and the mutual impedance to the vertical sides of the loop in Fig. 2(b) is therefore neglected.

2.2.2 The Current Distribution on the Support Structure

When the image of the support structure is considered together with the structure itself (as in Fig. 2(b)) it is seen that the radiating system is a rectangular loop; resonance may thus be expected when the perimeter, in wavelengths, is an integer. In addition, because the lower half of the loop is the ground reflexion of the upper half, the current distribution must be symmetrical about line g g' (Fig. 2(b)) and for resonance the perimeter must be an even number of wavelengths long. For the typical dimensions shown in Fig. 1(b) this leads to the conclusion that the only resonance which occurs in the h.f. band corresponds to a total loop perimeter of two wavelengths (i.e. a support structure perimeter of one wavelength).

Transmission line analogies can be used to obtain the current distribution along radiating systems and such an approximation is adequate for estimating the current distribution on the support structure. Thus the current distribution on the loop formed by the support structure and its image may be taken as

$$I(x) = I_s \cos\frac{2\pi x}{\lambda}, \quad -2L_1 < x < 2L_1, \qquad (5)$$

where λ = wavelength and $L_1 = \lambda/2$.

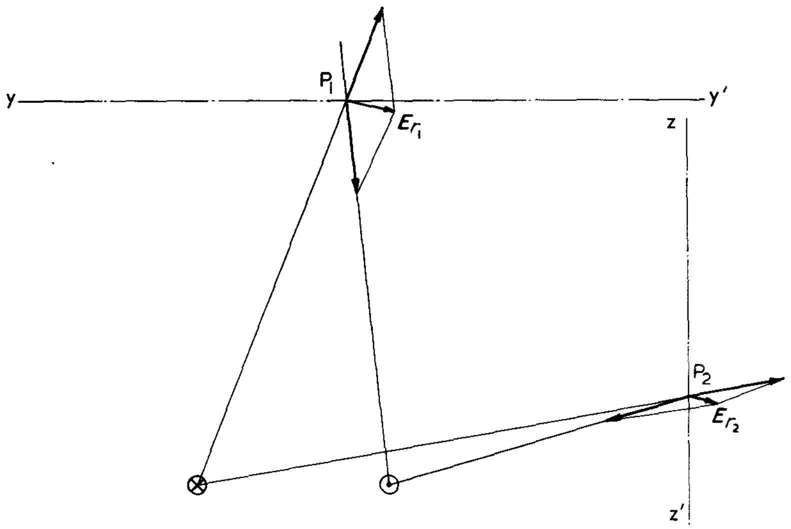

Fig. 3 - Method of obtaining the fields surrounding a two-conductor line

Putting expression (5) for $I(x)$ in equation (4) and evaluating $E(x)$ as outlined in Section 2.2.1, it is shown in Appendix I that the mutual impedance between the line and support structure in Fig. 2(a) is:

$$Z_M = 120 \frac{d}{D'} \; F(\zeta,\eta) \tag{6}$$

where

$$D' = \left[D^2 + (d/2)^2\right]^{\frac{1}{2}} \tag{7}$$

$$\zeta = D'/L \text{ and } \eta = 4L/\lambda \tag{8}$$

$F(\zeta,\eta)$ is an integral* which was evaluated by numerical methods and is plotted for a useful range of values in Fig. 4.

2.3 The Radiation Resistance of a Square Loop with a Perimeter of Two Wavelengths

In order to determine the power radiated by the support structure it is required to find the radiation resistance of the rectangular loop shown in Fig. 2(b). A general solution of this problem is excessively complicated, but in practice the ratio of the length to the breadth of the loop will be such that the radiation resistance will be approximately that of a square loop with the same perimeter.

A square loop with a perimeter of two wavelengths and with the current distribution shown in equation (5) may be regarded as made up of four half-wave dipoles with equal currents directed as shown in Fig. 5. The impedance of one dipole may therefore be readily determined from a knowledge of the self and mutual impedances and the result, derived in Appendix II, is 63·1 − j25·6 ohms. The reactive term is not important and merely means that resonance will occur at a

* see Appendix I, (16)

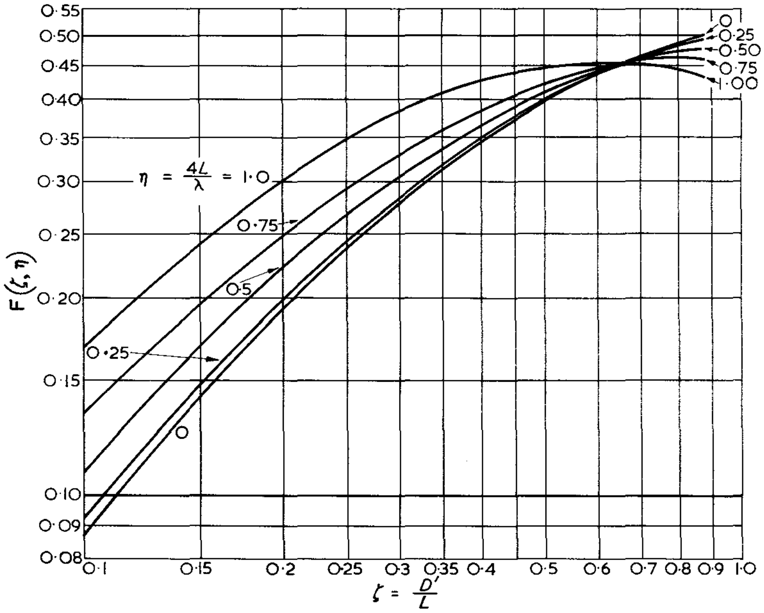

Fig. 4 - The function $F(\zeta,\eta) = \zeta \int_0^1 \frac{[\zeta^2 - \xi^2]}{[\zeta^2 + \xi^2]^2} \cdot \cos\frac{\xi\pi\eta}{2} \cdot d\xi$

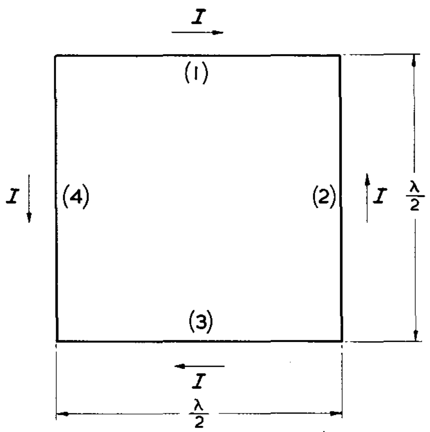

Fig. 5 - The currents in a square loop of side $\lambda/2$

frequency about 3% higher than expected. From the resistance term of a single dipole, it is seen that the total radiation resistance of the loop in Fig. 2(b) is 252 ohms and the radiation resistance R_s of the support structure in Fig. 2(a) is half this value, namely 126 ohms.

2.4 A Practical Example

Considering the practical structure shown in Fig. 1(b), resonance will occur when the sum of the lengths of the supports and cross piece is equal to one wavelength – that is, resonance occurs for a wavelength of 47 ft (14·3 m). D' is next evaluated and hence ζ and η: these have the values 0·275 and 0·64 respectively. $F(\zeta,\eta)$ from Fig. 4 is 0·30 and hence from equation (6):

$$Z_M = 120 \times \frac{1}{2{\cdot}062} \times 0{\cdot}30 = 17{\cdot}5 \text{ ohms}$$

The loss in power is therefore obtained from equation (2) and is, assuming

$Z_0 = 300$ ohms

$$\frac{(17{\cdot}5)^2}{126 \times 300} = 0{\cdot}0081, \text{ that is, } 0{\cdot}81\%$$

3. REINFORCED CONCRETE STRUCTURES

Reinforced concrete supports have been proposed as replacements for existing steel structures since they might cost less to maintain. Each member of one of these structures would consist of a number of steel rods embedded in a case of concrete. Because the transverse dimensions would be very small compared with a wavelength, the arrangement may be represented, for the purposes of analysis, by a single conductor of suitable length and cross section embedded in a cylinder of lossy dielectric.

Clothing a conductor in a lossy dielectric may be expected to produce three effects when compared with the behaviour of a solely metallic structure. First, the amplitude of the re-radiated field will be changed, secondly, power will be absorbed in the lossy dielectric, and thirdly, the electrical lengths of the various members of the structure will be altered and thus the resonant frequency will be changed. The magnitude of these effects depends on the electrical constants of the dielectric and the results of some experiments to determine the complex permittivity of concrete are given in Section 3.1.

It is shown in Section 3.2 that, even in the worst case, the power loss in the concrete is negligible while it is shown in Section 3.3 that as far as the amplitude of the re-radiation is concerned, the reinforced concrete behaves as if it consisted of the reinforcing rods only. Thus, the principal effect of the concrete is to alter the electrical length of the reinforcing members. This change in length is difficult to calculate but will be small and may be ignored for present purposes.

If there is no metallic connexion between the reinforcing rods of one member and those of another, a structure such as that in Fig. 1(b), made from reinforced concrete, would be equivalent to the system shown in Fig. 6. Resonance now occurs when the cross piece has a length of $\lambda/2$ and the radiation resistance R_s is approximately that of an isolated dipole, that is, 73 ohms. The mutual impedance Z_M is now given by equation (6) with η equal to unity and the appropriate value of ζ.

Thus, if the structure shown in Fig. 1(b) be made of reinforced concrete (without connexion between the reinforcing rods), resonance occurs at a wavelength of about 30 ft (9·1 m), corresponding to a frequency of about 33 Mc/s. The value of ζ is 0·275 as in Section 2.4 but that of η is unity and hence $F(\zeta,\eta)$, from Fig. 4, is 0·365. Substituting these values in equation (6) yields a value of 21·2 ohms for Z_M and hence from expression (2) the power loss for a line of Z_0 = 300 ohms is:

$$\frac{21{\cdot}2^2}{73 \times 300} \times 100\% = 2{\cdot}1\%$$

A frequency of 33 Mc/s is appreciably above the limits of the band normally used for h.f. broadcasting, but the use of a larger support structure than that shown in Fig. 1(b) is quite possible and the resonant frequency would then fall in the h.f. band.

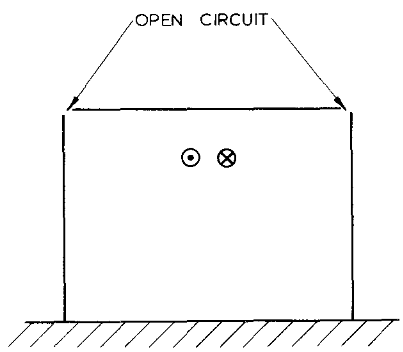

Fig. 6 - The metallic structure equivalent to a reinforced-concrete structure

The loss at resonance is appreciably greater than for the metallic structure considered in Section 2.4. Nevertheless, it would be possible to design the reinforced concrete structure so that metallic connexions could be made between the reinforcing rods of the members. In this case, the loss would be reduced to substantially the same value as in the example of Section 2.4 and would occur at approximately the same frequency.

3.1 Measurements of the Complex Permittivity of Concrete

The complex permittivity of concrete may be expected to vary from sample to sample and even within one sample. However, it is necessary to know at least the order of the quantities involved and measurements were therefore carried out on a block of reinforced concrete available from a dismantled reinforced concrete structure.

This block was traversed by two parallel iron reinforcing bars running through its length as shown in Fig. 7, and can be regarded as a short length of

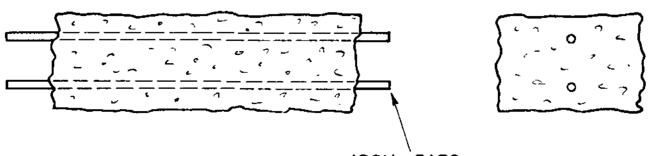

Fig. 7 - Reinforced concrete specimen used for permittivity measurements

concrete-loaded balanced line. Thus by measuring the open circuit and short circuit admittances of the line at various frequencies the average complex permittivity was obtained. Since the concrete is not of infinite extent, this method tends to give low values for the complex permittivity, but the error is not great because the fields round the rods are confined mainly within the concrete.

Measurements were carried out with the concrete in three conditions:

(a) thoroughly soaked in water

(b) dried out on a radiator for two days

(c) baked dry in an oven:

the results are summarized in Fig. 8. In practice the 'wet' curves will be applicable since the concrete will nearly always be saturated on account of moisture absorbed from the ground.

3.2 Power Lost in the Concrete

The order of the power loss in the concrete was assessed by consideration of the following simple model.

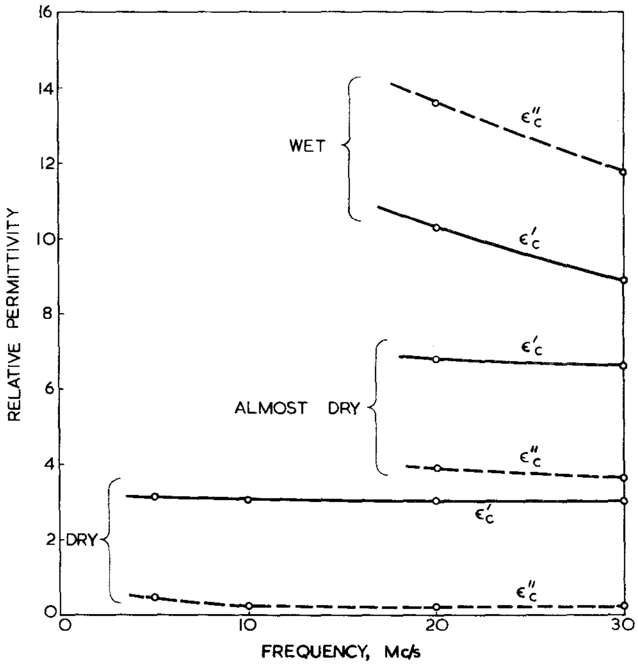

Fig. 8 - The electrical properties of concrete

A perfectly-conducting infinite sheet has a layer of lossy dielectric (complex permittivity $\epsilon_c = \epsilon_c' - j\epsilon_c''$) of width w next to it. A plane wave is incident on the lossy dielectric and the reflexion coefficient at the interface is found. This is compared with the reflexion coefficient of the conductor alone and the loss in the dielectric is thus the change in reflected power. The details of the calculation are given in Appendix III and it is shown that, provided that the width w of the concrete is a small fraction of a wavelength, the loss can be readily determined and is:

$$1434\left(\frac{w}{\lambda}\right)^2 \epsilon_c'' \quad \text{dB} \qquad (9)$$

This expression has a maximum value at the highest frequencies and if, for example, the width of the concrete is 2 in (5·1 cm) the loss is:

$$1434\left(\frac{2}{12 \times 30}\right)^3 \times 12 \text{ corresponding to } 0{\cdot}003 \text{ dB}$$

When the field from a transmission line takes the place of the plane wave assumed above and when a finite support structure is considered rather than an infinite sheet, it is clear that the fraction of power absorbed will be much less than the figures given above for the simple model.

3.3 The Power Reflected by an Infinite Conductor Surrounded by a Sleeve of Lossy Dielectric

Although the electric field produced by the transmission line is co-phased in the plane of the structure, its amplitude varies quite rapidly with distance and

a general solution of the concrete covered conductor is therefore difficult to obtain. However, for the purpose of comparing the behaviour of a conductor with a sleeve of lossy dielectric with that of a plain conductor it is permissible to consider that the incident wave is a plane wave and that the reflecting structure is an infinitely-long cylindrical perfect conductor surrounded by a sleeve of lossy dielectric as shown in Fig. 9.

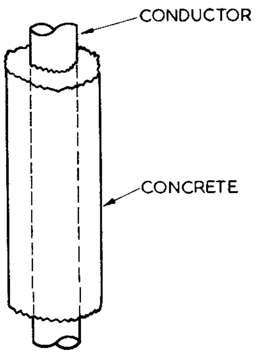

Fig. 9 - Infinite metallic rod clothed in concrete

This problem has been solved by Plonus;[4] from his analysis it is evident that where the radius of the composite structure is small compared with a wavelength, and where the dielectric has properties similar to those of concrete, it behaves as if it consisted of the inner perfect conductor alone. The physical explanation of this result is that, although the concrete is not a very good dielectric its properties are very much more like those of a dielectric than those of a conductor.

4. MULTICONDUCTOR TRANSMISSION LINES

The discussion in the previous sections has assumed a two-conductor transmission line but, although such lines are sometimes used, lines with four or more conductors are more common.

It would be possible to re-calculate the fields produced by the line by considering each conductor in turn and adding vectorially the various contributions to the field. However, this is unnecessary when it is not required to know the field very close to the conductors because conductors of like polarity are invariably grouped together and are equivalent to a single conductor of larger radius. Thus the "sausage" feeder shown in Fig. 10 having a characteristic impedance of 300 ohms, would produce the same scattered power from the support structure as the two-conductor line in Fig. 1(b) when d = 1 ft (0·31 m) and D = 2 ft (0·61 m).

5. CONCLUSIONS

A method has been presented by which the power lost from h.f. transmission lines due to the presence of supporting structures may be calculated. It is found that significant radiation (and hence loss) occurs only in a narrow band of frequencies when the support structure is resonant. For example, the loss due to the structure shown in Fig. 1(b) has a maximum value of 0·035 dB at 21 Mc/s representing a loss of 0·35 dB/1000 ft (1·15 dB/km) when the structures are placed at 100 ft (30·5 m) intervals. The total loss due to other causes in a four-conductor 6 s.w.g. (4·9 mm) 10 in × 6 in (25·4 cm × 15·2 cm) transmission line is[1] 0·45 dB/1000 ft (1·48 dB/km) at 21 Mc/s and hence it may be desirable to shift the frequency of the resonance (possibly by the addition of vertical or horizontal wires) if it is necessary to use the transmission line at this frequency.

Reinforced concrete members behave as if they consisted of the reinforcing rods only, but the resonance condition will be different if there is no electrical connexion between the rods of different members. In this case the power scattered

by a reinforced concrete structure is greater than that for the corresponding metal structure because the type of resonance is different. If there is a metallic connexion between the various reinforcing rods a reinforced concrete structure behaves approximately as an all-metal one, the only significant change being a slight alteration in the resonant frequency.

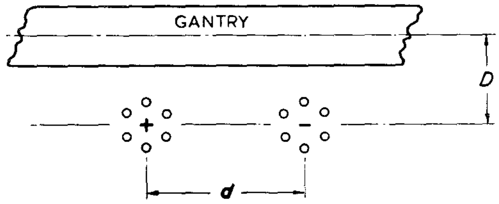

Fig. 10 - Multiconductor line

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APPENDIX I

Computation of the Mutual Impedance between Line and Support Structure

The mutual impedance required is given by the line integral round the path A-B-C in Fig. 2(b). That is:

$$Z_M = -\frac{1}{I_L}\int_{-L_1}^{L_1} E(x)\cos\frac{2\pi x}{\lambda}\,dx \tag{10}$$

where the terminals of the support structure are assumed to be at B in Fig. 2(b) and $E(x)$ includes the contribution of the image line. As shown in Section 2.2.1 the mutual impedance to the vertical sides of the loop may be neglected. Therefore:

$$\begin{aligned} Z_M &= -\frac{1}{I_L}\int_0^L E(x)\cos\frac{2\pi x}{\lambda}\,dx - \frac{1}{I_L}\int_{-L}^{0} E(x)\cos\frac{2\pi x}{\lambda}\,dx \\ &= -\frac{1}{I_L}\int_0^L E(x)\cos\frac{2\pi x}{\lambda}\,dx + \frac{1}{I_L}\int_{L}^{0} E(-x)\cos\frac{2\pi x}{\lambda}\,dx \\ &= -\frac{1}{I_L}\int_0^L [E(x)+E(-x)]\cos\frac{2\pi x}{\lambda}\,dx \end{aligned} \tag{11}$$

Because of the symmetry of the arrangement in Fig. 2(b), $E(-x)$ is equal to $E(x)$ and hence:

$$Z_M = -\frac{2}{I_L}\int_0^L E(x)\cos\frac{2\pi x}{\lambda}\,dx \tag{12}$$

In computing $E(x)$ the contribution from the image is now ignored because the resultant field falls off as the inverse square of the distance and $h >> D$. Using the method outlined in Section 2.2.1 the resultant component of the electric field of the line resolved along the horizontal member is:

$$E(x) = -60I_L\left[\frac{x+d/2}{D^2+(x+d/2)^2} - \frac{x-d/2}{D^2+(x-d/2)^2}\right]$$

which may be simplified to:

$$E(x) = -60I_L d\,\frac{[D'^2 - x^2]}{[D'^2+x^2]^2}\cdot\frac{1}{\left[1-\dfrac{x^2d^2}{[D'^2+x^2]^2}\right]} \tag{13}$$

where $D'^2 = D^2 + (d/2)^2$ (see Fig. 1(c)).

But $(d/2)^2 \ll D^2$ and equation (13) is therefore approximately:

$$E(x) = -\, 60 I_L d \, \frac{\left[D'^2 - x^2\right]}{\left[D'^2 + x^2\right]^2} \tag{14}$$

Now substitute the RHS of (14) in (12), to obtain:

$$Z_M = 120\, d \int_0^L \frac{\left[D'^2 - x^2\right]}{\left[D'^2 + x^2\right]^2} \cos \frac{2\pi x}{\lambda}\, dx$$

$$= 120 \frac{d}{D'}\, F(\zeta,\eta) \tag{15}$$

where $$F(\zeta,\eta) = \zeta \int_0^1 \frac{\left[\zeta^2 - \xi^2\right]}{\left[\zeta^2 + \xi^2\right]^2} \,.\, \cos \frac{\eta\pi\xi}{2} \,.\, d\xi \tag{16}$$

and $\zeta = D'/L$, $\eta = 4L/\lambda$ and $\xi = x/L$ (17)

$F(\zeta,\eta)$ has been evaluated numerically and is plotted in Fig. 4

APPENDIX II

The Radiation Resistance of a Square Loop of Side $\lambda/2$

The loop is shown in Fig. 5 with equal currents in the sides, directed as shown. Considering each side as a half-wavelength dipole the impedance of one side is:

$$Z = Z_{11} + \frac{I_2}{I_1} Z_{21} + \frac{I_3}{I_1} Z_{31} + \frac{I_4}{I_1} Z_{41} \tag{18}$$

Now
$$\frac{I_2}{I_1} = \frac{I_3}{I_1} = \frac{I_4}{I_1} = -1 \text{ and } Z_{21} = Z_{41}$$

$$\text{Hence } Z = Z_{11} - 2Z_{21} - Z_{31} \tag{19}$$

Z_{31} has been evaluated[5] and is equal to $-(12{\cdot}5 + j29{\cdot}5)$ ohms while Z_{11} is $73 + j42{\cdot}5$ ohms.[5] Z_{21} given by a formula derived by Murray[6] is equal to $11{\cdot}2 + j48{\cdot}8$ ohms.

$$\text{Hence } Z = 63{\cdot}1 - j25{\cdot}6 \text{ ohms} \tag{20}$$

The power radiated by each side of the loop is $63{\cdot}1\ I^2$ watts and hence the radiation resistance of the loop is 252 ohms. Correspondingly the radiation resistance of the support structure, assuming it to be above perfectly reflecting ground, is one half of this value, i.e. 126 ohms.

APPENDIX III

The Power Absorbed in a Slab of Lossy Dielectric Backed by a Perfect Conductor

The situation considered is that shown in Fig. 11. In the concrete ($-w<y<0$)

$$E_z = Ae^{\gamma_c y} + Be^{-\gamma_c y}$$

where A and B are constants and

$$\gamma_c = \gamma_0[\epsilon_c' - j\epsilon_c'']^{\frac{1}{2}} \quad , \quad \gamma_0 = j\omega[\mu_0\epsilon_0]^{\frac{1}{2}}$$

Subscript c refers to concrete, o to air.

Now $E_z = 0$ when $y = 0$

Therefore $E_z = A \sinh \gamma_c y$ (21)

and $H_x = (A/\eta_c) \cosh \gamma_c y$ (22)

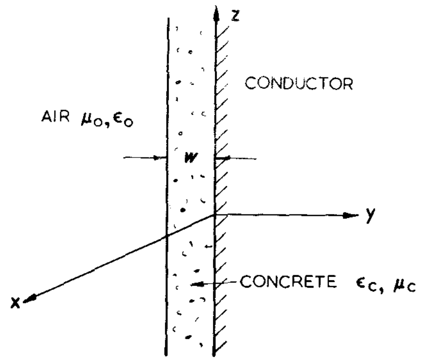

Fig. 11 - Model investigated in Appendix III

where $\eta = \eta_0 \ [\epsilon_c' - j\epsilon_c'']^{-\frac{1}{2}}$ and $\eta_0 = [\mu_0/\epsilon_0]^{\frac{1}{2}}$

Outside the concrete $y<-w$

$$E_z = Ce^{\gamma_0 y} + De^{-\gamma_0 y} \tag{23}$$

and

$$H_x = C/\eta_0 e^{\gamma_0 y} - D/\eta_0 e^{-\gamma_0 y} \tag{24}$$

At the concrete/air interface E_z and H_x are continuous and hence equations (21) and (23) may be equated when $y = -w$. Similarly equations (22) and (24) may be equated when $y = -w$ and from these resulting equations the ratio C/D may be found. That is:

$$\frac{C}{D} = \frac{1 - [\epsilon_c' - j\epsilon_c'']^{\frac{1}{2}} \coth \gamma_c w}{1 + [\epsilon_c' - j\epsilon_c'']^{\frac{1}{2}} \coth \gamma_c w} \tag{25}$$

When $\gamma_c w \ll 1$

$$\left|\frac{C}{D}\right| \approx 1 - \frac{2\theta_w^3}{3} \epsilon_c'' + 2\theta_w^2 \tag{26}$$

where $\theta_w = 2\pi w/\lambda$

When there is no concrete:

$$\left|\frac{C}{D}\right| \approx 1 + 2\,\theta_w^2 \tag{27}$$

Hence $$\frac{\text{Reflected wave (with concrete)}}{\text{Reflected wave (without concrete)}} \approx 1 - \frac{2\,\theta_w^3}{3}\,\epsilon_c'' \tag{28}$$

and thus the loss in power is:

$$20 \log \left[1 - \frac{2\,\theta_w^3}{3}\,\epsilon_c''\right] \quad \text{dB}$$

$$\approx 8{\cdot}68 \left[\frac{2\,\theta_w^3}{3}\right]\epsilon_c'' \quad \text{dB}$$

$$= 1454 \left(\frac{w}{\lambda}\right)^3 \epsilon_c'' \quad \text{dB} \tag{29}$$

LIST OF SYMBOLS

d = conductor spacing (two-conductor transmission line)

r_1 = distance from conductor to point on support structure under consideration

w = width of concrete slab

x = distance along perimeter of the support structure

D = distance between line and cross-piece of support structure

D' defined as $\left[D^2 + (d/2)^2\right]^{\frac{1}{2}}$

$E(x)$ = the electric field tangential to the support structure produced by the line

E_{r1}, E_{r2} = resultant electric fields produced by a transmission line

$F(\zeta, \eta)$ = an integral which is defined in Appendix I

h = height of transmission line above ground

I_s = current in support structure at the reference terminals

$I(x)$ = current in support structure as a function of x

I_L = current in transmission line

L = half the length of the cross-piece of the support structure

L_1 defined as $L + h + D$ (this is equal to $\lambda/2$ in the resonant case)

R_s = radiation resistance of support structure

Z = impedance of one limb of 2λ square loop

Z_{11} = self-impedance of a $\lambda/2$ dipole

Z_{12}, Z_{13}, Z_{14} = mutual impedances defined in Appendix II

Z_M = mutual impedance between line and support structure

Z_0 = characteristic impedance of line

Z_s = self-impedance of support structure

$\epsilon_c = \epsilon_c' - j\epsilon_c''$ = relative complex permittivity of concrete

ζ defined as D'/L

η defined as $4L/\lambda$

λ = wavelength in free space

ξ defined as x/L

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